Triangle Similarity Criteria

Two triangles are **similar** if their **corresponding angles are equal and their corresponding sides are in proportion**.

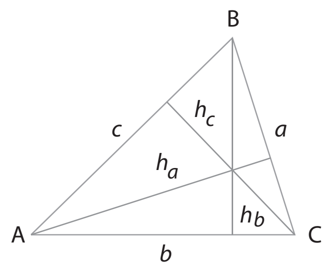
Let ABC and A'B'C' be two triangles. They are **similar** if one of the following holds:

- two corresponding angles are congruent;
- two corresponding sides are in proportion and the included angles are congruent;
- all three corresponding sides are in proportion.

Perimeter and Area of a Triangle

Perimeter

$$P = a + b + c$$



Area

$$A = \frac{ah_a}{2} = \frac{bh_b}{2} = \frac{ch_c}{2}$$

Heron's Formula

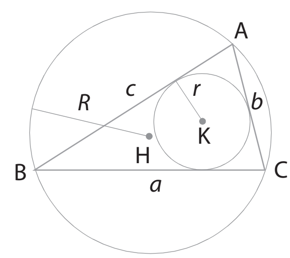
Heron's formula is used to find the area of any triangle when the lengths of its three sides are known:

$$A = \sqrt{p(p-a)(p-b)(p-c)}$$

where p is the semiperimeter (that is, $p = P/2$).
Moreover,

$$A = r \cdot p = \frac{a \cdot b \cdot c}{4R}$$

where r and R are the radii of the incircle and circumcircle.



Radius of the Incircle

$$r = \frac{A}{p} = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

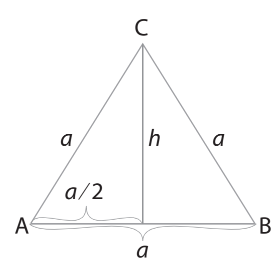
Radius of the Circumcircle

$$R = \frac{a \cdot b \cdot c}{4A}$$

Equilateral Triangles

Perimeter

$$P = 3a$$



▶ These formulas, and the ones that follow, use Pythagoras' theorem, which will be introduced later.

Altitude (or height)

$$h = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2}a = \frac{3}{2}R = 3r$$

Area

$$A = \frac{ah}{2} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{3}h^2 = \frac{3\sqrt{3}}{4}R^2 = 3\sqrt{3}\,r^2$$

where r and R are the radii of the incircle and circumcircle.

Radius of Incircle

$$r = \frac{h}{3} = \frac{\sqrt{3}}{6}a = \frac{R}{2}$$

Radius of the Circumcircle

$$R = \frac{2}{3}h = \frac{\sqrt{3}}{3}a = 2r$$